The affective domain in mathematical problem solving

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Summary

As a work based learning student and one of the coordinators of teaching Mathematics in primary education, I conducted a survey involving the 5th grade pupils of five schools in Paphos (two urban and three rural) , having as my main aim the investigation of the affective domain in mathematical problem solving. The questionnaire, the interview and the observation were used as the main methods of collecting my data. Overall, the fifth graders appeared to be influenced by the affective variables: self-efficacy, anxiety-phobia, pleasure and the type of the problem in problem solving. No significant differences were noted in the affective domain relevant to the type of school. On the contrary, differences were indicated relevant to the type of the problem, the school performance and the gender of the pupils. Additionally, the research revealed some beliefs which are consistent to the documented literature, whereas some others come in contrast with previous researches. Consequently, the affective domain has a vital role in mathematical problem solving and further research is needed in this field.

Mathematical problem solving - Affective domain

A problem is only a problem (as mathematicians use the word) if you don't know how to go about solving it. A problem that has no "surprises" in store, and can be solved comfortably by routine or familiar procedures (no matter how difficult!) is an exercise. (Schoenfeld, 1983)

Affect serves as an extraordinarily powerful evolutionary language for communication. This system functions through 'body language', eye contact, facial expressions, tone of voice, as well as spoken language, cries, laughter etc. (Goldin, cited in Leder et al., 2002)

Certain **sub domains (variables)** of the affective representation can be distinguished: (1). **emotions** (rapidly changing states of feeling, mild to very intense), (2) a**ttitudes**

(moderately stable predispositions towards ways of feelings in classes of situations, involving a balance of affect and cognition), and (3) **beliefs** (internal representations to which the holder attributes truth validity, or applicability, usually stable and highly cognitive).

Aim and objectives of the project

The aim of the project was to examine ways in which the affective domain relates to and influences mathematical problem solving. In more detail my objectives were the following:

- To discover the emotions, beliefs and attitudes of 5th grade pupils when they deal with a mathematical problem and to explore the relationships of these affective variables to the effective Mathematical Problem Solving (MPS).
- To assess the group differences between male and female pupils regarding their emotions, beliefs and attitudes related to MPS.
- To define the differences regarding the emotions, beliefs and attitudes of pupils studying in different types of school.
- To estimate the differences of the domain variables according to the problem type.

Methodology

It was through hours of reflection, literature review and discussions with my colleagues that I came to the conclusion that the most appropriate main method for me to carry out my study would be a survey. Since, my intention was to gather information about the affective domain in MPS, within a relatively wide range of the 5th grade Cypriot pupils this method would enable me to succeed this goal. Actually my sample consisted of 177 5th grade pupils from rural and urban primary schools in Paphos.

The questionnaire enabled me to gather data from a relatively wide range of respondents. A pilot questionnaire was used. The final questionnaire was composed of thirty one Likert-type scale questions (closed questions) on emotions, beliefs and attitudes of 5th grade pupils. The results were analysed by the statistics software package SPSS (12). The statistical treatment of the data reflected some inferential analysis (eg. Correlation, regression etc).

However in an attempt to provide depth to my research, add quality to my data and increase confidence in my results, I also conducted interviews with ten pupils. The interviews were semi-structured, with a flexible schedule. They included the set of themes, presented on the questionnaire with the flexibility of following some useful emerging issues. Interviewing permitted me to ask for clarification and much elucidation of opinions. The pupils, who were interviewed, were also asked to solve four problems (two routine and two non-routine problems). While they were solving the problems, they were analysing their thoughts loudly and their reactions were observed and noticed. These observations were noticed on a 'Mood Map' (Appendix 1) which I developed. The data from interviews – observations were analysed thematically, based on the coding scheme I developed through the quantitative analysis.

Additionally, further triangulation was pursued by using documents and literature such as academically based-books, journals, policy focused documents (of the school policy) etc.

Project Findings

By using exploratory factor analysis I identified underlying 'factors' that explain the correlations between the items of the questionnaire. The Principal Components analysis which creates linear combinations of the observed factors was used as extraction method. A four factor model derived since it was found that four Eigen values were greater than 1. Moreover, 54.3% of the total variance is attributable to the first four factors and this means that a model with four factors could be considered as adequate to represent the data. These factors are: 'Type of the problem', 'Self-efficacy', 'Anxiety-phobia' and 'Pleasure'. The fifth dimension of the scheme is relevant to the factor 'beliefs' which is another variable of the affective domain in mathematical problem solving.

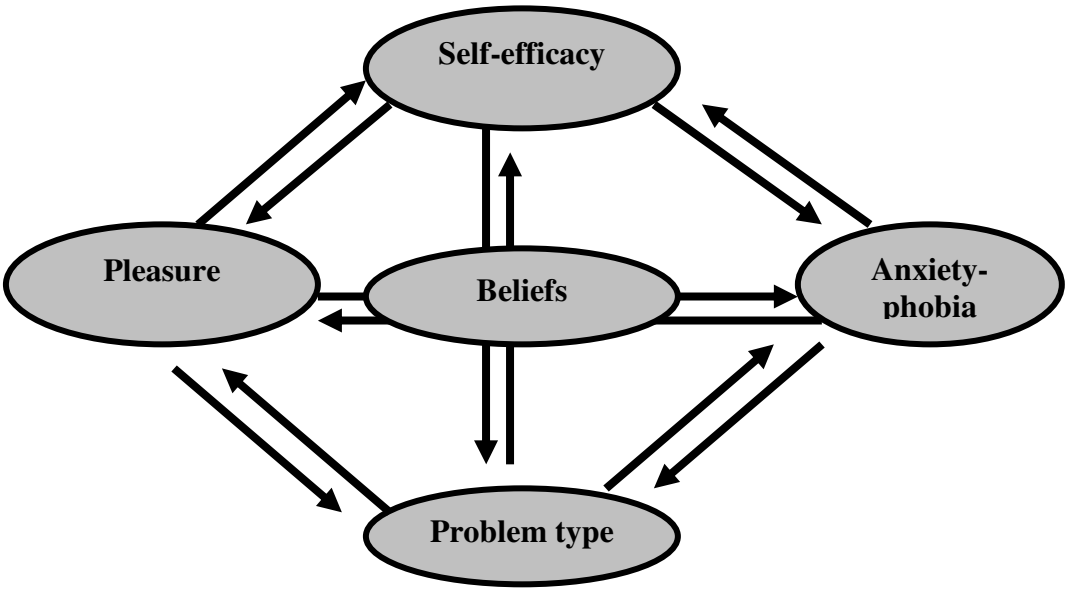

Figure 1: The affective domain's coding scheme

Some of the results of my research are in line with previous researchers whereas others come in contrast. In general, the 5th grade pupils expressed a positive attitude towards mathematical problem solving (self-efficacy) as well as negative attitudes. A high correlation between negative emotions (anxiety-phobia) and self-efficacy (attitudes) as well between positive emotions (pleasure) and self-efficacy (positive attitude) was found. The high school performance pupils seem to state a more positive affective domain to mathematical problem solving than the low school performance pupils. These results strongly agree with the results of the qualitative analysis.

The research did not reveal any statistically significant differences between the pupils who study in urban schools and the pupils, who study in rural schools, as well as between males and females. The type of the problem also influences the affective domain of the problem solver, but the correlation is not as strong enough as the correlation between emotions and attitudes.

The 5th grade pupils with special educational needs expressed negative attitudes, emotions and beliefs regarding to mathematical problem solving but this results cannot be generalised since only two SEN took part in the research.

Conclusions

The main beliefs expressed by the 5th grade pupils regarding mathematical problem solving were the following:

1. If they cannot solve a mathematical problem in ten minutes or less, they will not be able to solve it at all.
2. There is not only one way to solve a mathematical problem (usually the rule that the teacher has most recently demonstrated to class).
3. One mathematical problem has more than one solution.
4. Only the smart pupils can solve mathematical problems.
5. Memorising mathematical rules is the only way to solve quickly and correctly a mathematical problem.
6. Mathematical problem solving has nothing to do with real life situations (neither positive nor negative statements).
7. When they cooperate with the rest members of their teams, they always solve correctly a mathematical problem

The 2nd and the 3rd belief come in contrast with similar researches in the past since they had the opposite results. In more detail, the 5th grade pupils (64.9 %) stated that there is not only one way to solve a mathematical problem. It is also worth mentioning that they hold the belief that one mathematical problem, has more than one solution. According to an extended list of typical pupils' beliefs derived from literature the opposite were of the strong beliefs of the pupils. This finding implies that Cypriot pupils have experienced much more prototype problems than pupils in other countries. That partly can be explained due to the use of the new Cypriot mathematics textbooks in which the pupils are frequently confronted with problems which have more than one solution and can be approached in a number of ways. Through interviewing, pupils stated that there are many ways of solving a problem.

Recommendations

The results of this study can be used, at organisational level as well as at national level, in order to accomplish better results in the affective domain in problem solving process. As one of the coordinators in teaching Mathematics in my school I recommended my colleagues to cultivate the self-efficacy of their pupils by giving them opportunities of

success in problem solving, to give them support and encouragement when their development slows or appears to regress, to present them accessible problems which are easily understood and problems that can be solved in a number of ways etc.

Reflection

Conducting this research gave me the chance to reflect on the dynamics of the mutual influence between myself and my workplace. I experienced a much more effective research than the traditional one and I learned how to develop a more ethical concept of my research and my role as a teacher in general. I became a much more effective work based researcher and a self managed learner.

Appendix 1: The Mood Map

Name	Characteristics	Routine problem 1	Routine problem 2	Non-routine problem 1	Non-routine problem 2
Anna	♀, HP, U	☺ ☼ !*	☺ ☼ !*	☺♦ ■ ! *	☺ ■ ♥!*
Marios	♂, HP, U	☺☼ ♥ ! *	☺☼ ♥ !*	☺☼ ► !*	☺☼♥ ►! *
Eleni	♀, LP, U	▬ ● ☼ ∩ ◄	▬ ● ○ Ω ▼	▬ ■ ╬ ▼	▬ ●▼
Stelios	♂, LP, U	☺ ☼ ! *	☺☼ ! *	♦ ☼ ■ ♣ ▼	♦ ■ ♣ ▼
Christina	♀, HP, R	☼ ■ ►*!	☼■►* !	▬ ● ○∩☼!	▬● ╬ ▼☼
Kyriakos	♂, HP, R	☼ ■ ● * !	☺ ☼ ♥ * !	☺ ☼ ♥ * !	☺▬●╬ ▼
Maria	♀, LP, R	♦ ╬ ☼ * !	☼ ○▬ ╬ ▼	▬ ○■ ▼	▬ ○■ ▼
Nikolas	♂, LP, R	☺ ♦ ■ * !	☺ ►♥ !	☺▬ ∩ ╬	▬ ☼►!
Tina	♀, LP, R (SEN)	☺ ♦ ♣ ▼	♦ ♣ ▼	☼ ♥ !	■ ♦ ♣ ▼
George	♂, LP, U (SEN)	♦ ▬ ▼	▬ ▼	☼►♥!	☺ ☼ ► !

U= urban school
R= rural school
♂= Male
♀= Female
HP= High school Performance
LP= Low school performance
SEN= Special Educational Needs
*= success in the solution of the problem
! = Satisfaction
Ω= despair
∩= blocked
● = eating his/her fingers
○ = hands shaking
╬ = confused

♥ = pleasure
☺= self-efficacy
☼ = calm
♦ = indifference
♣ = boredom
▬ = Pessimism
■ = Impatience
►= Interested
▼= gave up
◄= crying